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Direct Solution of the Aeroelastic Stability Equations

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Introduction

CURRENT aeroelastic analyses are performed through repetitive solutions of a complex eigenvalue problem or a determinant for a large number of reduced frequencies or velocities.[1,2] The stability of the system is studied by numerically tracing the roots of the solutions in the traditional V-g-ω plots. The critical modes are then obtained from the crossing of the zero line of the real part of the roots (zero damping or flutter condition) and the imaginary part of the roots (zero frequency or divergence condition). The variants of the solutions can be separated into three main groups, namely the 1) p methods, 2) the k methods, and 3) the p-k methods. Many authors[3] have discussed the difference between these traditional solutions. While all these methods lead to approximately the same flutter and divergence speeds, they differ in the results obtained beyond the critical speeds. Classically, the display of the results in the form of V-g-ω diagrams is made on the implicit assumption that the shape of these plots reveal something about the severity of the instability beyond the critical speeds. Such interpretation must be made with great caution, since some of the solution variants, e.g., the classical k methods, can produce erroneous conclusions if interpretation about the damping obtained beyond the critical speeds is made.[3] In many practical applications the main objective is the determination of the critical speeds and not the damping beyond these speeds. Examples are preliminary design and optimal design programs subjected to constraints on the critical aeroelastic velocities. In such conditions a rapid and automatic method for the evaluation of the least flutter and divergence speeds is required. In Ref. 4, it has been demonstrated that this end can be achieved when only in-quadrature air forces are considered in the nonstationary aerodynamic load formulation. In this Note, it is shown that in many cases where out-of-phase air loads are also considered, the problem can be cast into the solution of two equations, whose simultaneous solution determines directly the critical velocities and frequencies. Such applications include the formulations based on quasisteady aerodynamic theory and simple forms of unsteady aerodynamic loads.

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Problem Formulation

The aeroelastic stability equations can be written in general form as

$$[p^2[M] + [K] - \Lambda[a(p)]]\{u\} = \{0\} \tag{1}$$

where $[M]$ is the mass matrix, $[K]$ is the stiffness matrix, Λ is an aerodynamic parameter function of the freestream velocity, and p is the problem eigenvalue, which is in general complex. The elements of the aerodynamic matrix, which are explicit functions of p, will depend on the theory used for the formulation of unsteady aerodynamic loads. In many cases, e.g., when the quasisteady aerodynamic theory or simple forms of unsteady aerodynamic loads are used, the matrix $[a(p)]$ assumes the simple form

$$[a(p)] = \gamma p[a_1] + [a_0] \tag{2}$$

where γ is an aerodynamic damping parameter, and the aerodynamic matrices $[a_1]$ and $[a_0]$ are real, with constant elements. Substituting Eq. (2) into Eq. (1), transforming the coordinates to the free vibration modal base, and using mode shapes normalized for the unit mass matrix, the aeroelastic stability equations read

$$[-p^2[I] + [\xi^2] - \Lambda[A_0] - i\gamma\Lambda p[A_1]]\{q_0\} = \{0\} \tag{3}$$

where solutions in the form

$$\{q\} = \{q_0\}e^{ipt} \tag{4}$$

have been considered for the modal amplitudes. Equation (3) represents a parametric eigenvalue problem whose characteristic equation can be written in general form as

$$p^{2n} + ic_1p^{2n-1} + c_2p^{2n-2} + \cdots + c_{2n} = 0 \tag{5}$$

where n is the order of the system. Since the matrices of Eq. (3) are all real, it follows that the coefficients of the characteristic Eq. (5) are real and are functions of the aerodynamic parameter Λ and the damping factor γ. Now, on the borderline of stability, the roots p of Eq. (5) are real, so that we can separate Eq. (5) into two equations for the real and the imaginary parts to read

$$k^{2n} + c_2 k^{2n-2} + \cdots + c_{2n} = 0 \tag{6}$$

$$c_1 k^{2n-1} + c_3 k^{2n-3} + \cdots + c_{2n-1} k = 0 \tag{7}$$

where k is real. The static divergence condition is obtained from Eq. (6) as

$$c_{2n} = 0 \tag{8}$$

The flutter condition is then obtained by solving Eqs. (6) and (7) for $k \neq 0$, which are two equations for two unknowns, namely, the flutter velocity V_F and the flutter frequency ω_F. The coefficients c_i of Eqs. (6) and (7) often assume simple forms, and this is shown in the applications presented in the next section.

Applications

Aeroelasticity of Plates and Shells Using Quasisteady Aerodynamics

Aeroelasticity of plates and shells is an important branch of flight vehicles flutter analysis in supersonic flow regime.[1,5,6] The quasisteady aerodynamic theory has been used in almost all analytical researches on the subject. According to this theory, the nonstationary aerodynamic pressure difference Δp is related to the panel transverse displacement w by an expression in the form[1,5]

$$\Delta p = -\frac{2Q}{(M^2-1)^{1/2}}\frac{\partial w}{\partial x} - \frac{2Q(M^2-2)}{V(M^2-1)^{3/2}}\frac{\partial w}{\partial t} \tag{9}$$

where Q is the freestream dynamic pressure, M is the freestream Mach number, and V is the freestream velocity. Using Eq. (9), the stability equations in the modal base, with modes normalized to unit generalized mass, can be written as

$$[-k^2[I] + [\xi^2] - \lambda[A] - i\lambda gk[I]]\{q_0\} = \{0\} \tag{10}$$

where λ and g are the dynamic pressure and aerodynamic damping parameters. It can be shown by expansion that the coefficients of Eqs. (6) and (7) can be expressed as

$$c_m = \sum_j^L \frac{d_{(m-j)/2}(\lambda g)^j e_j}{j!} \tag{11}$$

where

$$j = 0, 2, 4, \ldots \quad \text{and} \quad L = \frac{m+2}{2}$$

for m even

$$j = 1, 3, 5, \ldots \quad \text{and} \quad L = \frac{m+1}{2}$$

for m odd

$$e_j = \prod_{i=1}^{j} \frac{(2n+2i-m-j)}{2}$$

for $j = 1, 2, 3, \ldots$

$$e_j = 1 \quad \text{for} \quad j = 0 \tag{12}$$

The coefficients d_i in Eq. (11) are obtained from the characteristic equation coefficients when only in-quadrature aerodynamic loads are considered,[4] and are expressed in terms of the traces of the power products of the $[A]$ and $[\xi^2]$ matrices. As a numerical application we consider a simply supported flat square plate of dimension a, thickness h, material mass density ρ, and flexural rigidity D. The dynamic pressure parameter λ and the aerodynamic damping parameter g of Eq. (10) for this case are

$$\lambda = \frac{8Qa^3}{D\pi^4(M^2-1)^{1/2}}$$

$$g = \frac{M^2-2}{M^2-1}\frac{\pi^2}{Va}\sqrt{\frac{D}{\rho h}} \tag{13}$$

and the aeroelastic eigenvalue k is related to the flutter frequency ω through the relation

$$k^2 = (\rho h a^4 / D\pi^4)\omega^2 \tag{14}$$

Using a four-mode solution with a halfsine wave in the cross stream direction and a number of halfsine waves equal to 1, 2, 3, and 4 in the streamwise direction, the coefficients c_i can be determined in terms of g and λ. The flutter condition can then be obtained by solving Eqs. (6) and (7) simultaneously for different values of the damping factor. Notice the simplification introduced by the present formulation, where all the flutter velocities and corresponding flutter frequencies are obtained directly from the solution of these two equations. Furthermore, the present formulation can provide first the least instability, i.e., the first flutter velocity, if this is the prime factor of the analysis. Figure 1 gives the results of the numerical calculations performed for different values of the damping factor g. From these curves it can be observed that the inclusion of aerodynamic damping for this type of problem is always conservative. Furthermore, for high values of damping, higher modes can be more critical than the first ones, even for this simple problem, and therefore a convergence study of the solution must be made by including more modes in the analysis when high values of aerodynamic damping is present. Finally, it is to be observed that practical values of g^2 range between 0–0.03.

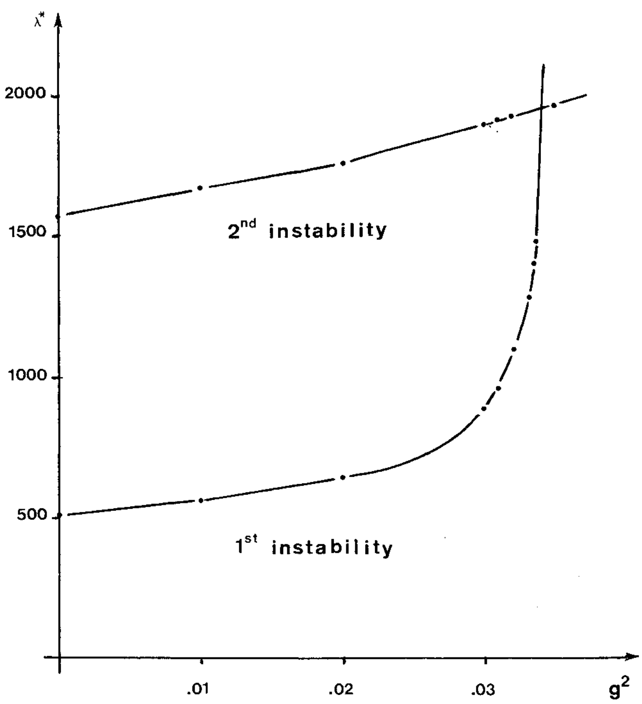

Fig. 1 Flutter dynamic pressure vs aerodynamic damping parameter of a flat square plate.

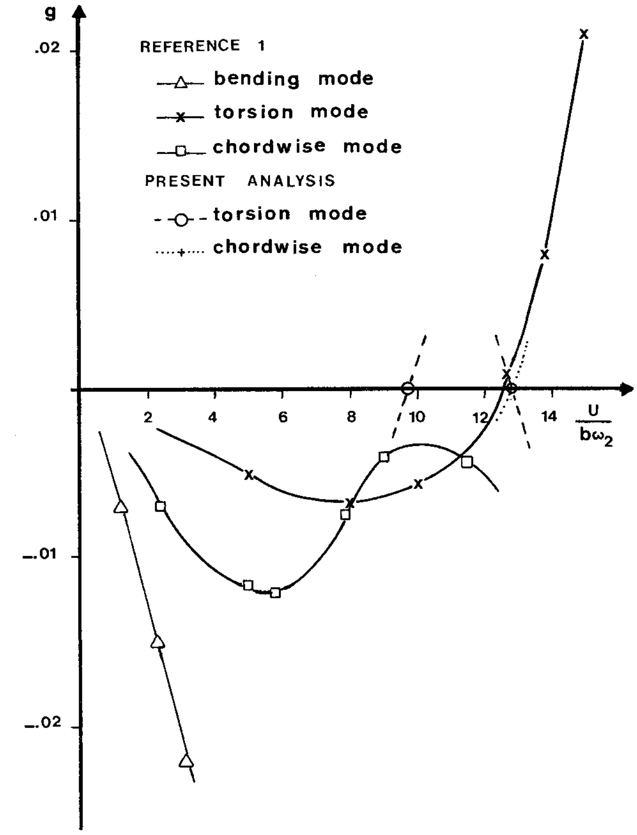

Fig. 2 Aeroelastic stability of a low aspect-ratio cantilever wing.

Hypersonic Flutter of a Cantilever Wing

Consider the hypersonic flutter of the cantilever wing studied in Ref. 7 and reported in Ref. 1. This model has been analyzed with great details in Ref. 4 using in-quadrature air loads. The pioneering and now classical work of Ref. 7, made well before the advent of high computational devices, was performed using the classical k method of flutter analysis. The main purpose of analyzing this example here is to show that the traditional V-g-ω plots using the k or p-k methods can miss some mild mode instabilities, since the solution is made at discreet values of reduced frequencies or velocities. The data of the model are given in the cited references. We consider only one case of Λ_3 (the ratio of chordwise to torsion frequency), namely, $\Lambda_3 = 1.833$, in this section. The results of the analysis are shown in Fig. 2. As can be seen from this figure, the problem presents two critical modes, the first one being a mild bending-torsion flutter mode, and the second being a violent torsion-chordwise flutter mode. The present formulation shows that the first instability occurs for the mild mode at a value of $U/b\omega_2 = 9.72$ and a flutter frequency ratio $\omega/\omega_2 = 0.7058$. This mode becomes stable again at a value of $U/b\omega_2 = 12.92$ and a frequency ratio of 0.643. The second mode becomes unstable at a value of $U/b\omega_2 = 12.97$ and a frequency ratio of 1.273. The analysis of Ref. 7 made for discrete values of reduced frequency jumps the first instability and only detects the second one.

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Prevention of Jump in Inertia-Coupled Roll Maneuvers of Aircraft

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Nomenclature

I_x, I_y, I_z	=	resp. roll, pitch, and yaw inertia moments
i_1, i_2, i_3	=	cyclically $(I_z - I_y)/I_x$, etc.
l, m, n	=	resp. roll, pitch, and yaw moment coefficients
p, q, r	=	resp. roll, pitch, and yaw rates
y, z	=	resp. side and normal force coefficients
α, β	=	resp. angle of attack and sideslip
$\delta a, \delta e, \delta r$	=	resp. aileron, elevator, and rudder deflection

Subscript

$\alpha, p, \ldots$	=	stability derivative w.r.t. $\alpha, p, \ldots$

Superscripts

^	=	indicates division by one of i_1, i_2, i_3
	=	indicates transpose

Introduction

THE problem of inertia coupling in rapidly rolling aircraft was first recognized when divergent motions were predicted by Phillips[1] at certain critical roll rates. These critical roll rates were obtained by Rhoads and Schuler[2] as steady-state solutions of simplified equations that neglected gravity terms. Such solutions obtained by neglecting the weight components in body axes are called pseudosteady state (PSS) solutions. The PSS method was employed by Schy and Hannah[3] for a coupled, nonlinear system of equations for the aircraft dynamics with a linear aerodynamic model to represent the equilibrium solutions as a function of a control parameter. Jump was shown to occur for a critical value of the control parameter at which a turning point (also termed a saddle-

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